Sample Question Paper 3

Mathematics 10th (Standard)

Time : 3 Hrs. **Max. Marks : 80**

General Instructions

Read the following instructions carefully and follow them.

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into 5 section A, B, C, D and E.
3. In **Section A,** question numbers 1-18 are Multiple Choice Questions (MCQs) and question number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In **Section B,** question numbers 21-25 are Very Short Answer (VSA) type questions, carrying 02 marks each.
5. In **Section C,** question numbers 26-31 are Short Answer (SA) type questions, carrying 03 mark each.
6. In **Section D,** question numbers 32-35 are Long Answer (LA) type questions, carrying 05 marks each.
7. In **Section E,** question numbers 36-38 are Case Study Based questions carrying 4 marks each with sub parts of the values 1, 1 and 2 marks each respectively.
8. There is no overall choice. However, an internal choice in 2 questions of section B, 2 questions of section C and 2 questions of section D has been provided. An internal choice has been provided in all the 2 marks questions of section E.
9. Draw neat and clear figures wherever required. Take $\pi = 22/7$ wherever required if not stated.
10. Use of calculators is not allowed.

Section A Multiple Choice Questions (Each Que. carries 1 M)

1. If HCF of 91 and 117 is 13, then their LCM is

(a) 819 (b) 130 (c) 117 (d) 273

2. If $\cot^2 45° - \sin^2 30° = p \cdot \sin 30° \cos 60°$, then the value of p is

(a) 4 (b) $\frac{1}{2}$ (c) 3 (d) $\frac{1}{3}$

3. The ratio of the total surface area to the lateral surface area of a cylinder with base radius 80 cm and height 20 cm is

(a) 1 : 2 (b) 2 : 1 (c) 3 : 1 (d) 5 : 1

4. The probability of getting 101 marks in out of 100 marks is

(a) 1 (b) $\frac{1}{2}$ (c) 0 (d) 2

5. The ratio in which the line $3x + y - 9 = 0$ divides the segment joining the points (1, 3) and (2, 7) is

(a) 3 : 4 (b) 4 : 3 (c) 2 : 3 (d) 3 : 2

6. If median = 137 and mean = 137.05, then the value of mode is

(a) 156.90 (b) 136.90 (c) 186.90 (d) 206.90

7. How many terms are there in the arithmetic series $1 + 3 + 5 + ... + 73 + 75$?

(a) 28 (b) 30 (c) 36 (d) 38

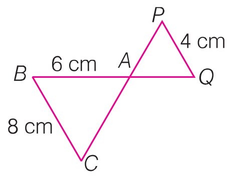
8. If $8\tan\theta = 15$, then the value of $\sin\theta - \cos\theta$ is

(a) $\frac{7}{17}$ (b) $\frac{17}{7}$ (c) $\frac{15}{17}$ (d) $\frac{8}{17}$

9. The distance of point $P(10, 12)$ from origin is

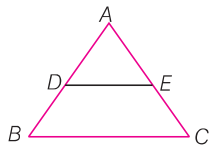
(a) 22 (b) 2 (c) $2\sqrt{61}$ (d) $\sqrt{44}$

10. In the given figure, if $\Delta ACB \sim \Delta APQ$, $BA = 6$ cm, $BC = 8$ cm and $PQ = 4$ cm, then the length of AQ is



(a) 3 cm (b) 5 cm (c) 4 cm (d) 6 cm

11. In the given figure, $DE \parallel BC$ and $\frac{AD}{DB} = \frac{3}{5}$. If $AC = 4.8$ cm, then the length of AE is



(a) 1.5 cm (b) 1.8 cm (c) 2 cm (d) 4.2 cm

12. If radius of a circle is 3 cm and tangent drawn from an external point to the circle is 4 cm, then the distance from centre of circle to the external point is

(a) 3 cm (b) 2 cm (c) 5 cm (d) 4 cm

13. Length of an arc of a sector of angle 120° of a circle with radius 21 cm is

(a) 44 cm (b) 63 cm (c) 66 cm (d) 110 cm

14. If probability of an event E is 0.3, then the probability of complement of event E is

(a) 0.3 (b) 0.7 (c) 0.2 (d) 0.15

15. The sum of two numbers x and y, $x > y$ is 137 and their difference is 43. This situation can be algebraically represented as

(a) $x - y = 137$, $x + y = 180$
(b) $2(x + y) = 137$, $2(x - y) = 43$
(c) $x + y = 137, x - y = 43$
(d) $x + y = 43$, $x - y = 137$

16. The LCM of the smallest composite number and the smallest prime number is

(a) 2 (b) 4 (c) 6 (d) 8

17. If α and β are the zeroes of the polynomial $f(x) = x^2 - p(x+1) - c$ such that $(\alpha+1)(\beta+1) = 0$, then c is equal to

(a) 1 (b) 0 (c) −1 (d) 2

18. Length of the shadow of a tree 15 m long when the Sun's angle of elevation is 30°, is

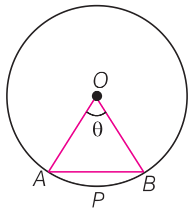
(a) 15° (b) $15\sqrt{3}$ (c) $5\sqrt{3}$ (d) 7.5

Assertion-Reason Based Questions

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion** (A) In the figure, O is the centre of a circle. If the area of sector $OAPB$ is $\frac{9}{36}$ of the area of the circle, then the value of θ is 90°.



Reason (R) Length of an arc of a circle with radius r and central angle θ is given by $l = \frac{\theta}{360°} \times \pi r^2$.

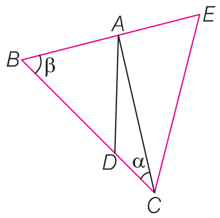
20. **Assertion** (A) $\sqrt{2}$ is an irrational number.

Reason (R) If p be a prime, then $\sqrt{p}$ is an irrational number.

Section B Very Short Answer Type Questions (Each Que. carries 2 M)

21. In the given figure, $\frac{BC}{BD} = \frac{BE}{AC}$ and $\angle\alpha = \angle\beta$.

Show that $\Delta ABD \sim \Delta EBC$.



22. For what value of p, will the following system of linear equations represent parallel lines?

$-x + py = 1$ and $px - y = 1$

23. A chord of a circle of radius 35 cm subtends a right angle at the centre. Find the area of minor segment.

24. (a) Prove that $(\tan^2 A - \tan^2 B) = \frac{(\sin^2 A - \sin^2 B)}{\cos^2 A \cos^2 B} = \frac{\cos^2 B - \cos^2 A}{\cos^2 B \cos^2 A}$.

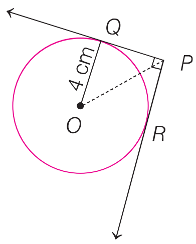
Or

(b) If $\sqrt{3}\tan 2\theta - 3 = 0$, then find the value of $\cos\theta$.

25. (a) A line through the centre O of a circle of radius 5 cm cuts the tangent at a point P on the circle at point Q such that $OQ = 13$ cm. Find the length of PQ.

Or

(b) In the given figure, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle QPR = 90°$, then find the length of PQ.



Section C Short Answer Type Questions [Each Que. carries 3 M]

26. Show that $4-\sqrt{5}$ is irrational.

27. The king, queen and jack of clubs are removed from a deck of 52 playing cards and then well-shuffled. Now, one card is drawn at random from the remaining cards. Find the probability of getting a card of

(i) a diamond

(ii) a queen

28. (a) Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Or

(b) The radii of two concentric circles are 26 cm and 16 cm. AB is a diameter of the bigger circle. BD is a tangent to the smaller circle touching it at D. Find the length AD.

29. (a) Given that the sum of the zeroes of the polynomial $(a+1)x^2+(2a+3)x+(3a+4)$ is -1. Find the product of its zeroes.

Or

(b) Represent the quadratic polynomial x^2-x-12 on the graph paper and find the zeroes.

30. Use elimination method to find all possible solutions of following pair of linear equations $px+qy=-p-q$ and $qx-py+p+q=0$.

31. Prove that $\dfrac{(1+\cot A+\tan A)(\sin A-\cos A)}{(\sec^3 A-\operatorname{cosec}^3 A)}=\sin^2 A\cos^2 A.$

Section D Long Answer Type Questions [Each Que. carries 5 M]

32. (a) From a solid cylinder whose height is 12 cm and diameter is 10 cm, a conical cavity of same height and same diameter is hollowed out. Find the volume and total surface area of the remaining solid.

Or

(b) A right angled triangle whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone, so formed. [choose the value of π as found appropriate]

33. Ram and Shyam together have 55 marbles. Both of them lost 5 marbles each and the product of the number of marbles they have is 164. Find out how many marbles they had to start with?

34. (a) The mean of the following frequency table is 50 but the frequencies f_1 and f_2 in class interval 20-40 and 60-80 are missing. Find the missing frequencies.

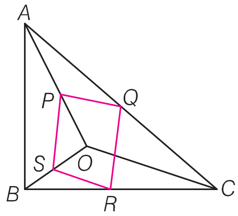
Class interval	0-20	20-40	40-60	60-80	80-100	Total
Frequency	17	f_1	32	f_2	19	120

Or

(b) Compute the median from the following data.

Mid-value	115	125	135	145	155	165	175	185	195
Frequency	6	25	48	72	116	60	38	22	3

35. In the given figure, if $PQRS$ is a parallelogram and $AB \parallel PS$, prove that $OC \parallel SR$.



Section E Case-Study/Passage-Based (Each Que. carries 4 M)

36. Isha is planning to grow her orchard. She wants to plant rows of fruit trees in a way that each row has more trees than the one before, following a specific pattern. Given below are the details of her plants.

(a) The first row will have 5 trees.

(b) Each new row will have 3 more trees than the one before.

(c) There will be a total of 10 rows of trees.

Based on the above information, answer the following questions.

(i) Calculate the number of trees in the 10th row of the orchard. **[1]**

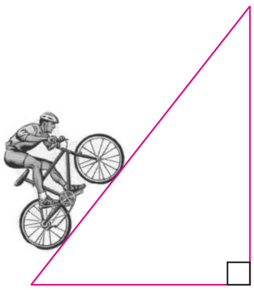
(ii) Find the ratio of number of trees in 5th and 6th row. **[1]**

(iii) (a) What will be the total number of trees in the orchard after all 10 rows are planted? Also, find the number of rows, if total number of trees is 258. **[2]**

Or

(b) Isha changed her plan by not planting in rows 5 and 6 to create a pathway for walking without altering the pattern for the rows. All rows will have the same number of trees as before. Calculate the number of trees now. **[2]**

37. A cyclist is climbing through a 20 m long rope which is highly stretched and tied from the top of a vertical pole to the ground as shown below



Based on the above information, answer the following questions.

(i) Find the height of the pole, if angle made by rope with the ground level is 60°. **[1]**

(ii) (a) If the angle made by the rope with the ground level is 45°, then find the height of the pole. **[2]**

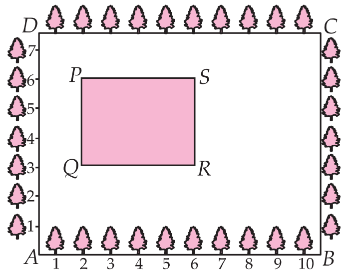
Or

(b) If the angle made by the rope with the ground level is 45° and 3 m rope is broken, then what will be the height of the pole. **[2]**

(iii) If the angle made by the rope with the ground level is 60°, then calculate the distance between artist and pole at ground level. **[1]**

38. Tree Plantation to Control Pollution

The class X students of a secondary school in Krishnagar have been alloted a rectangular plot of land for this gardening activity Sapling of Gulmohar are planted on the boundary at a distance of 1 m from each other.



There is a rectangular gracy lawn in the plot as shown in the figure. The students sowing seeds of flowering plants on the remaining area of the plot.

Based on the above information, answer the following questions.

(i) Find the coordinates of point Q and S. **[1]**

(ii) (a)If the point M divides the line QS in the ratio 3 : 2, then find the coordinates of M. **[2]**

Or

(b) If the point G divides the line QR in the ratio 1 : 2, then find the coordinates of G. **[2]**

(iii) Find the distance between the vertices of diagonal Q and S. **[1]**

Solutions

1. (a) Given, HCF (91, 117) = 13

We know that

$$\text{LCM} \times \text{HCF} = \text{Product of two numbers}$$

$$\Rightarrow \quad \text{LCM} \times 13 = 91 \times 117 \quad [\because \text{HCF} = 13]$$

$$\Rightarrow \quad \text{LCM} = \frac{91 \times 117}{13}$$

$$\Rightarrow \quad \text{LCM} = 819$$

2. (c) We have, $\cot^2 45° - \sin^2 30° = p \sin 30° \cdot \cos 60°$

$$\Rightarrow \quad (1)^2 - \left(\frac{1}{2}\right)^2 = p\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\left[\because \sin 30° = \cos 60° = \frac{1}{2}, \cot 45° = 1\right]$$

$$\Rightarrow \quad 1 - \frac{1}{4} = \frac{p}{4} \Rightarrow \frac{3}{4} = \frac{p}{4}$$

$$\Rightarrow \quad p = 3$$

3. (d) Given, radius of the base, $r = 80$ cm

and height of the cylinder, $h = 20$ cm

$$\therefore \frac{\text{Total surface area}}{\text{Lateral surface area}} = \frac{2\pi r(h + r)}{2\pi rh} = \frac{h + r}{h} = \frac{20 + 80}{20} = \frac{100}{20} = \frac{5}{1} = 5 : 1$$

4. (c) In out of 100 marks, we do not get 101 marks, it is an impossible event. And the probability of an impossible event is 0.

Therefore, the probability of getting 101 marks in out of 100 marks is zero.

5. (a) Suppose the line $3x + y - 9 = 0$ divides the line segment joining points $A(1, 3)$ and $B(2, 7)$ in the ratio $k : 1$ at point C.

Then, coordinates of C are $\left(\frac{2k + 1}{k + 1}, \frac{7k + 3}{k + 1}\right)$.

[by Section formula]

But C lies on the line $3x + y - 9 = 0$.

Therefore, $3\left(\frac{2k + 1}{k + 1}\right) + \frac{7k + 3}{k + 1} - 9 = 0$

$$\Rightarrow \quad (6k + 3) + (7k + 3) - 9k - 9 = 0 \Rightarrow 4k - 3 = 0$$

$$\Rightarrow \quad k = \frac{3}{4}$$

So, the required ratio is 3 : 4 internally.

6. (b) Given, median = 137 and mean = 137.05

We know that

$$\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$$
$$= 3(137) - 2(137.05)$$
$$= 411 - 274.10 = 136.90$$

7. (d) Given, arithmetic series is

$$1+3+5+\ldots+73+75$$

Here, first term $a=1$, common difference $d=2$

and last term $=t_n=75$

Let number of terms in given series be n.

Then, $t_n=a+(n-1)d$

$\Rightarrow$ $75=1+(n-1)\times 2$

$\Rightarrow$ $75=1+2n-2 \Rightarrow 2n=76$

$\Rightarrow$ $n=38$

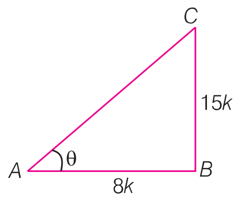
Hence, there are 38 terms in given series.

8. (a) We have, $\tan\theta=\dfrac{15}{8}$

Let us draw right angled ΔABC, and $\angle CAB=\theta$

Now, $\tan\theta=\dfrac{15}{8}$

$\Rightarrow$ $\dfrac{BC}{AB}=\dfrac{15}{8}$ $\left[\because \tan\theta=\dfrac{\text{Perpendicular}}{\text{Base}}\right]$



Let $AB=8k$ and $BC=15k$

In right angled ΔABC, using Pythagoras theorem,

$$AC=\sqrt{(AB)^2+(BC)^2}$$
$$=\sqrt{(8k)^2+(15k)^2}$$
$$=\sqrt{64k^2+225k^2}=\sqrt{289k^2}=17k$$

Now, $\sin\theta=\dfrac{BC}{AC}=\dfrac{15k}{17k}=\dfrac{15}{17}$ $\left[\because \sin\theta=\dfrac{\text{Perpendicular}}{\text{Hypotenuse}}\right]$

and $\cos\theta=\dfrac{AB}{AC}=\dfrac{8k}{17k}=\dfrac{8}{17}$ $\left[\because \cos\theta=\dfrac{\text{Base}}{\text{Hypotenuse}}\right]$

$\therefore$ $\sin\theta-\cos\theta=\dfrac{15}{17}-\dfrac{8}{17}=\dfrac{7}{17}$

9. (c) Given, points are $P(10, 12)$ and origin $O(0, 0)$.

Here, $(x_1, y_1)=(10, 12)$ and $(x_2, y_2)=(0, 0)$

Distance between two points

$$=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

[by distance formula]

$\therefore$ $OP=\sqrt{(0-10)^2+(0-12)^2}$

$$=\sqrt{100+144}$$
$$=\sqrt{244}=2\sqrt{61}\text{ units}$$

10. (a) Given, $\Delta ACB \sim \Delta APQ$

Since, corresponding sides of similar triangle are proportional.

So, $\dfrac{AB}{AQ}=\dfrac{BC}{QP}$

$\Rightarrow$ $\dfrac{AB}{BC}=\dfrac{AQ}{QP}$

$\Rightarrow$ $\dfrac{6}{8}=\dfrac{AQ}{4}$

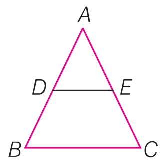
[$\because$ $AB=6$cm, $BC=8$cm and $PQ=4$cm, (given)]

$\Rightarrow$ $\dfrac{6\times 4}{8}=AQ$

$\therefore$ $AQ=3$cm

11. (b) Let $AE=x$ cm

Then, $EC=(AC-AE)=(4.8-x)$ [$\because$ $AC=4.8$, given]



Now, in ΔABC, $DE \parallel BC$

$\therefore$ $\dfrac{AD}{DB}=\dfrac{AE}{EC}$

[basic proportionality theorem]

$\Rightarrow$ $\dfrac{3}{5}=\dfrac{x}{4.8-x}$

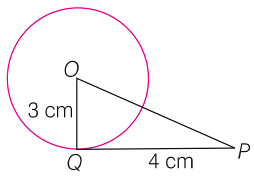
$\Rightarrow$ $3(4.8-x)=5x$

$\Rightarrow$ $8x=14.4 \Rightarrow x=1.8$

Hence, $AE=1.8$ cm

12. (c) Let O be the centre of the circle and PQ be the tangent drawn from an external point P to the circle.

$\therefore OQ=3$ cm and $PQ=4$ cm.



Here, $OQ \perp PQ$

[$\because$ tangent to a circle is perpendicular to the radius through the point of contact]

In right angled ΔOQP, using Pythagoras theorem,

$$OP=\sqrt{(OQ)^2+(QP)^2}$$
$$=\sqrt{(3)^2+(4)^2}=\sqrt{9+16}=\sqrt{25}=5\text{ cm}$$

13. (a) Given, angle of sector, $\theta=120^\circ$

and radius of circle, $r=21$cm

We know that length of an arc $=\dfrac{\theta}{360^\circ}\times 2\pi r$

$\therefore$ Required length of arc $=\dfrac{120^\circ}{360^\circ}\times 2\pi\times 21$

$$=\frac{1}{3}\times 2\times\frac{22}{7}\times 21$$
$$=2\times 22=44\text{ cm}$$

14. (b) Given, probability of event E, $P(E) = 0.3$

We know that sum of probability of an event and probability of its complement is 1.

$\therefore$ Probability of complement of event $E = P(\overline{E}) = 1 - P(E)$

$\Rightarrow \quad P(\overline{E}) = 1 - 0.3 = 0.7$

15. (c) Given, two numbers are x and y, where $x > y$.

$\because$ Sum of numbers $= 137$

$\therefore \quad x + y = 137$

and difference of numbers $= 43$

$\Rightarrow \quad x - y = 43$

16. (b) Smallest composite number $= 4 = 2 \times 2 = 2^2$

and smallest prime number $= 2$

$\therefore$ LCM $(4, 2) = 2^2 = 4$

17. (a) Given, α and β are the zeroes of the polynomial

$$f(x) = x^2 - p(x+1) - c$$
$$= x^2 - px - p - c = x^2 - px - (p+c)$$

$\therefore \quad \alpha + \beta = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\dfrac{(-p)}{1} = p$

and $\quad \alpha\beta = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$= \frac{-(p+c)}{1} = -(p+c)$$

Now, it is given that $(\alpha + 1)(\beta + 1) = 0$

$\Rightarrow \quad \alpha\beta + \alpha + \beta + 1 = 0$

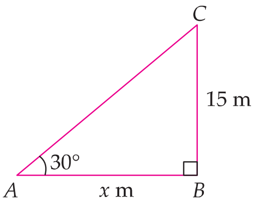
$\Rightarrow \quad -(p+c) + p + 1 = 0 \Rightarrow -c + 1 = 0$

$\therefore \quad c = 1$

18. (b) Let the length of shadow, $AB = x$ m.

We have, height of tree, $BC = 15$ m

and angle of elevation $= 30°$



In right angled ΔABC,

$$\tan 30° = \frac{BC}{AB}$$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{15}{AB} \qquad \left[\because \tan 30° = \dfrac{1}{\sqrt{3}}\right]$

$\Rightarrow \quad AB = 15\sqrt{3}$ m

Hence, the length of the shadow is $15\sqrt{3}$ m.

19. (c) Given, area of sector $OAPB = \dfrac{9}{36}$ of the area of circle.

We know that area of sector $= \dfrac{\theta}{360°} \times \pi r^2$

$\therefore \quad \dfrac{\theta}{360°}\pi r^2 = \dfrac{9}{36} \times \pi r^2 \quad [\because \text{area of circle} = \pi r^2]$

$\Rightarrow \quad \dfrac{\theta}{360°} = \dfrac{9}{36}$

$\Rightarrow \quad \theta = 90°$

Therefore, Assertion is true.

And we know that length of arc $= \dfrac{\theta}{360°} \times 2\pi r$

Therefore, Reason is false.

20. (a) We know that if p is a prime number, then $\sqrt{p}$ is an irrational number.

Here, 2 is a prime number.

$\therefore \sqrt{2}$ is an irrational number.

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

21. In ΔABC, $\quad \angle\alpha = \angle\beta \quad$ [given]

$\therefore \quad AB = AC$

[$\because$ sides opposite to equal angles of a triangle are also equal]

Also, given that $\quad \dfrac{BC}{BD} = \dfrac{BE}{AC}$

$\Rightarrow \quad \dfrac{BC}{BD} = \dfrac{BE}{AB} \qquad [\because AB = AC]$ ...(i) **[1]**

In ΔABD and ΔEBC, we have

$\angle EBC = \angle ABD \qquad$ [common]

and $\quad \dfrac{BC}{BD} = \dfrac{BE}{AB} \qquad$ [from Eq. (i)]

$\therefore$ By SAS similarity criterion, we get

$\Delta ABD \sim \Delta EBC \qquad$ **Hence Proved. [1]**

22. Given, pair of equations is

$-x + py - 1 = 0 \qquad$ ...(i)

and $\quad px - y - 1 = 0 \qquad$ ...(ii)

On comparing the given equations with standard form i.e. $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$a_1 = -1, b_1 = p, \quad c_1 = -1$

and $\quad a_2 = p, \ b_2 = -1, c_2 = -1$

For parallel lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \underset{\text{I}}{\frac{-1}{p}} = \underset{\text{II}}{\frac{p}{-1}} \neq \underset{\text{III}}{\frac{-1}{-1}} \qquad \text{...(iii)}$$ **[1]**

On taking I and II terms, we get

$$\frac{-1}{p} = \frac{p}{-1} \Rightarrow p^2 = 1 \Rightarrow p = \pm 1$$

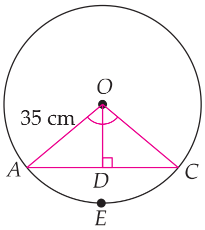
Since, $p = -1$ does not satisfy the last two terms of Eq. (iii).

$\therefore \quad p = 1$ is the required value.

Hence, for $p = 1$, the given system of equations will represent parallel lines. **[1]**

23. Given, radius of circle, $AO = OC = 35$ cm

and AC is a chord such that $\angle AOC = 90°$



Area of $\Delta AOC = \frac{1}{2} \times OA \times OC$

[$\because$ Area of triangle $= \frac{1}{2} \times$ Base $\times$ Perpendicular]

$= \frac{1}{2} \times 35 \times 35 = \frac{1225}{2}$ cm^2

$= 612.5$ cm^2 **[1]**

Area of sector $OAECO = \frac{\theta}{360°} \times \pi r^2 = \frac{90°}{360°} \times \frac{22}{7} \times 35 \times 35$

$= 962.5$ cm^2

Area of minor segment $AECDA$ = Area of sector $OAECO$ − Area of ΔAOC

$= 962.5 - 612.5 = 350$ cm^2 **[1]**

24. (a) LHS $= \tan^2 A - \tan^2 B = \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$

$\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$= \frac{\sin^2 A \cos^2 B - \cos^2 A \sin^2 B}{\cos^2 A \cos^2 B}$

$= \frac{\sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B}{\cos^2 A \cos^2 B}$

$[\because \cos^2\theta = 1 - \sin^2\theta]$

$= \frac{\left[\sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B\right]}{\cos^2 A \cos^2 B}$

$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$ = Mid-term

[1]

$= \frac{(1-\cos^2 A) - (1-\cos^2 B)}{\cos^2 A \cos^2 B}$ $[\because \sin^2\theta = 1 - \cos^2\theta]$

$= \frac{1 - \cos^2 A - 1 + \cos^2 B}{\cos^2 A \cos^2 B} = \frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B}$

= RHS **Hence proved. [1]**

Or

(b) We have,

$\sqrt{3}\tan 2\theta - 3 = 0 \Rightarrow \sqrt{3}\tan 2\theta = 3$

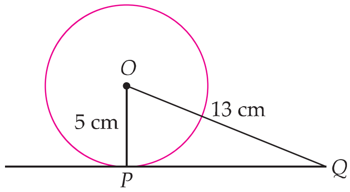
$\Rightarrow \tan 2\theta = \frac{3}{\sqrt{3}} \Rightarrow \tan 2\theta = \sqrt{3}$

$\Rightarrow \tan 2\theta = \tan 60°$ $[\because \tan 60° = \sqrt{3}]$ **[1]**

$\Rightarrow 2\theta = 60° \Rightarrow \theta = \frac{60°}{2} = 30°$

$\therefore \cos\theta = \cos 30° = \frac{\sqrt{3}}{2}$ **[1]**

25. (a) We know that tangent at a point on a circle is perpendicular to the radius through the point of contact.



[1]

Therefore, OP is perpendicular to PQ.

In right angled ΔOPQ, we have

$OQ^2 = OP^2 + PQ^2$ [by Pythagoras theorem]

$\Rightarrow PQ^2 = OQ^2 - OP^2$

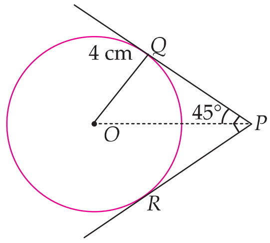
$\Rightarrow PQ^2 = 13^2 - 5^2$

$\Rightarrow PQ^2 = 169 - 25 = 144$

$\Rightarrow PQ = 12$ cm **[1]**

Or

(b) We know that if pair of tangents are drawn from an external point P, then line joining from centre O to the point P, bisects the angle at P.



$\therefore \angle OPQ = \frac{\angle QPR}{2} = \frac{90°}{2} = 45°$ **[1]**

Also, radius of circle OQ is perpendicular to the tangent line QP.

Now, in right angled ΔOQP,

$\tan 45° = \frac{OQ}{QP} \Rightarrow 1 = \frac{4}{QP}$ $[\because OQ = 4 \text{ cm}, \tan 45° = 1]$

$\Rightarrow QP = 4$ cm

Hence, length of PQ is 4 cm. **[1]**

26. Let us assume that $(4-\sqrt{5})$ is a rational number. Then, it will be of the form $\frac{p}{q}$, where p, q are co-prime integers and $q \neq 0$.

Now, $4 - \sqrt{5} = \frac{p}{q}$ **[1]**

On rearranging, we get

$4 - \frac{p}{q} = \sqrt{5}$

Since, 4 and $\frac{p}{q}$ are rational numbers. So, their difference will be a rational number.

$\therefore \sqrt{5}$ is rational. **[1]**

But we know that $\sqrt{5}$ is irrational.

So, this contradicts the fact that $\sqrt{5}$ is irrational.

Therefore, our assumption is wrong.

Hence, $4-\sqrt{5}$ is irrational. **[1]**

27. Here, king, queen and jack of clubs are removed from a deck of 52 playing cards.

So, remaining cards in deck $= 52 - 3 = 49$

$\therefore$ Total number of outcomes $= 49$ **[1]**

(i) We know that there are 13 cards of diamonds.

Let E_1 = Event of getting a diamond

Then, number of outcomes favourable to $E_1 = 13$

$\therefore$ Required probability $= P(E_1) = \frac{13}{49}$ **[1]**

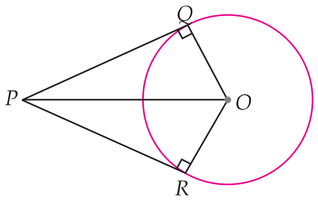
(ii) We know that there are 4 queens in a deck. After removing a queen of club, we are left with 3 queens.

Let E_2 = event of getting a queen.

Then, number of outcomes favourable to $E_2 = 3$

$\therefore$ Required probability $= P(E_2) = \frac{3}{49}$ **[1]**

28. (a) Let PQ and PR be two tangents drawn from an external point P to a circle with centre O.



To prove $\angle QOR + \angle QPR = 180°$

Proof In ΔOQP and ΔORP,

$PQ = PR$ [$\because$ tangents drawn from an external point are equal in length] **[1]**

$OQ = OR$ [radii of circle]

$OP = OP$ [common sides]

$\therefore \quad \Delta OQP \cong \Delta ORP$ [by SSS congruence rule]

Then, $\angle QPO = \angle RPO$ [by CPCT]

and $\angle POQ = \angle POR$ [by CPCT]

$\Rightarrow \quad \angle QPR = 2\angle QPO$
and $\angle QOR = 2\angle QOP$ $\Big\}$ ...(i) **[1]**

Now, in right angled ΔOQP,

$\angle QPO + \angle QOP + \angle OQP = 180°$

[by angle sum property of a triangle]

$\Rightarrow \quad \angle QPO + \angle QOP = 90°$ [$\because \angle OQP = 90°$]

$\Rightarrow \quad \angle QOP = 90° - \angle QPO$

$\Rightarrow \quad 2\angle QOP = 180° - 2\angle QPO$

[multiplying both sides by 2]

$\Rightarrow \quad \angle QOR = 180° - \angle QPR$ [from Eq. (i)]

$\Rightarrow \quad \angle QOR + \angle QPR = 180°$ **Hence proved. [1]**

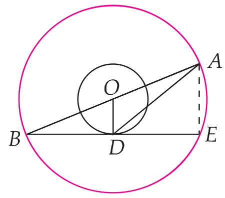
Or

(b) Let the line BD intersects the bigger circle at E.

Now, Join AE.

Let O be the centre of the bigger circle, then O is the mid-point of AB.

[$\because AB$ is a diameter of the bigger circle]



[1]

BD is a tangent to the smaller circle and OD is a radius through the point of contact D.

Then, $\quad OD \perp BD \Rightarrow OD \perp BE$

Since, OD is perpendicular to the chord BE of bigger circle.

$\therefore \quad BD = DE$

[$\because$ perpendicular drawn from the centre to a chord bisects the chord]

$\Rightarrow$ D is the mid-point of BE.

In ΔBAE, O is the mid-point of AB and D is the mid-point of BE.

$\therefore \quad OD = \frac{1}{2} AE$

[$\because$ line segment joining the mid-points of any two sides of a triangle is half of the third side]

$\Rightarrow \quad AE = 2(OD) = 2 \times 16 = 32$ cm

[$\because$ radius of smaller circle, $OD = 16$ cm] **[1]**

In right angled ΔODB, using Pythagoras theorem,

$OD^2 + BD^2 = OB^2$

$\Rightarrow \quad BD = \sqrt{OB^2 - OD^2} = \sqrt{26^2 - 16^2}$

[$\because$ radius of bigger circle, $OB = 26$ cm]

$= \sqrt{676 - 256} = \sqrt{420}$

$\therefore \quad DE = BD = \sqrt{420}$

In right angled ΔAED, use Pythagoras theorem, we have

$AD = \sqrt{(AE)^2 + (DE)^2}$

$= \sqrt{(32)^2 + (\sqrt{420})^2} = \sqrt{1024 + 420}$

$= \sqrt{1444} = 38$ cm **[1]**

Value Point

For two concentric circles, if chord of bigger circle BE is a tangent to smaller circle and OD is radius through the point of contact D, then OD always divides BE in two equal parts.

29. (a) Given, the sum of the zeroes of the quadratic polynomial $(a+1)x^2+(2a+3)x+(3a+4)$ is -1.

$$\therefore \quad \frac{-(2a+3)}{a+1} = -1$$ **[1]**

$$\left[\because \text{Sum of zeroes} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}\right]$$

$$\Rightarrow \quad a+1 = 2a+3 \Rightarrow a = -2$$ **[1]**

Now, product of zeroes $= \dfrac{3a+4}{a+1} = \dfrac{3(-2)+4}{-2+1} = 2$

$$\left[\because \text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}\right]$$ **[1]**

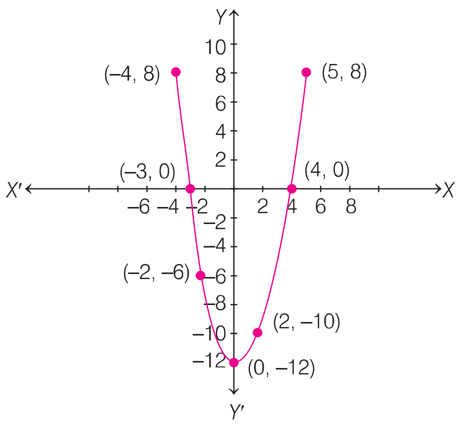
Or

(b) Let $y = x^2 - x - 12$.

To draw its graph, we need the values of y corresponding to different values of x. For this consider, the following table.

x	0	2	4	5	-4	-3	-2
y	-12	-10	0	8	8	0	-6

Now, plot all these points on graph. **[1]**



[1]

Thus, we get the graph of $x^2 - x - 12$, which is a parabola opening in upward direction and it intersects the X-axis at two points whose coordinates are -3 and 4.

Hence, -3 and 4 are the zeroes of the given quadratic polynomial. **[1]**

30. Given, pair of linear equation is

$$px + qy = -p - q \qquad \text{...(i)}$$

and $$qx - py + p + q = 0 \qquad \text{...(ii)}$$

On multiplying Eq. (i) by 'p' and Eq (ii) by q, we get

$$p^2x + pqy = -p^2 - pq \qquad \text{...(iii)}$$

and $$q^2x - pqy + pq + q^2 = 0 \qquad \text{...(iv)}$$

On rearranging the Eq. (iii), we get

$$p^2x + pqy + p^2 + pq = 0 \qquad \text{...(v)}$$

On adding Eqs. (iv) and (v), we get

$$p^2x + q^2x + 2pq + p^2 + q^2 = 0$$

$$\Rightarrow \quad x(p^2+q^2) = -(p^2+q^2+2pq)$$

$$\Rightarrow \quad x = \frac{-(p+q)^2}{p^2+q^2}$$ **[1½]**

On putting $x = \dfrac{-(p+q)^2}{p^2+q^2}$ in Eq. (i), we get

$$p\cdot\left[\frac{-(p+q)^2}{p^2+q^2}\right] + qy = -p - q$$

$$\Rightarrow \quad qy = -p - q + \frac{p(p+q)^2}{p^2+q^2}$$

$$\Rightarrow \quad y = \frac{(p+q)}{q}\left[\frac{p(p+q)}{p^2+q^2} - 1\right]$$

$$\Rightarrow \quad y = \left(\frac{p+q}{q}\right)\left[\frac{p^2+pq-p^2-q^2}{p^2+q^2}\right]$$

$$\Rightarrow \quad y = \frac{(p+q)}{q}\left[\frac{pq-q^2}{p^2+q^2}\right]$$

Hence, the possible solutions are

$$x = \frac{-(p+q)^2}{p^2+q^2},$$

$$y = \frac{(p+q)}{q}\left[\frac{pq-q^2}{p^2+q^2}\right].$$ **[1½]**

31. LHS $= \dfrac{(1+\cot A + \tan A)(\sin A - \cos A)}{(\sec^3 A - \text{cosec}^3 A)}$

$$= \frac{\left(1 + \dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\cos A}\right)(\sin A - \cos A)}{\left(\dfrac{1}{\cos^3 A} - \dfrac{1}{\sin^3 A}\right)}$$

$$\left[\begin{array}{l}\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}, \cot\theta = \dfrac{\cos\theta}{\sin\theta} \\ \text{and } \sec\theta = \dfrac{1}{\cos\theta}, \text{cosec}\,\theta = \dfrac{1}{\sin\theta}\end{array}\right]$$ **[1]**

$$= \frac{\left(1 + \dfrac{\cos^2 A + \sin^2 A}{\sin A \cos A}\right)(\sin A - \cos A)}{\dfrac{\sin^3 A - \cos^3 A}{\sin^3 A \cos^3 A}}$$

$$= \frac{\left(1 + \dfrac{1}{\sin A \cos A}\right)(\sin A - \cos A)}{\dfrac{\sin^3 A - \cos^3 A}{\sin^3 A \cos^3 A}}$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$ **[1]**

$$= \left(\frac{\sin A \cos A + 1}{\sin A \cos A}\right)\cdot\frac{\sin^3 A \cos^3 A \times (\sin A - \cos A)}{\sin^3 A - \cos^3 A}$$

$$= \frac{(\sin A \cos A + 1)}{\sin A \cos A} \cdot \frac{(\sin^3 A \cos^3 A) \times (\sin A - \cos A)}{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}$$

$[\because a^3 - b^3 = (a-b)(a^2 + b^2 + ab)]$

$$= \frac{(\sin A \cos A + 1)(\sin^2 A \cos^2 A)}{\sin^2 A + \cos^2 A + \sin A \cos A}$$

$$= \frac{(\sin A \cos A + 1)(\sin^2 A \cos^2 A)}{1 + \sin A \cos A} = \sin^2 A \cos^2 A$$

$[\because \sin^2 \theta + \cos^2 \theta = 1]$

= RHS **Hence proved. [1]**

32. (a) Given, diameter of the cylinder, $d = 10 \text{ cm}$

$\Rightarrow$ Radius, $r = 5 \text{ cm}$

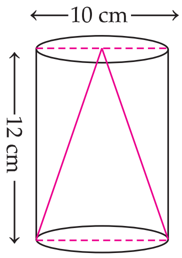
and height of the cylinder, $h = 12 \text{ cm}$

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 5 \times 5 \times 12 = \frac{6600}{7} \text{ cm}^3$$ **[1]**

and volume of the cone $= \frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12 = \frac{2200}{7} \text{ cm}^3$$ **[1/2]**



Hence, volume of remaining solid

= Volume of the cylinder – Volume of the cone

$$= \frac{6600}{7} - \frac{2200}{7} = \frac{4400}{7} = 628.57 \text{ cm}^3$$ **[1]**

Now, slant height of the cone,

$$l = \sqrt{r^2 + h^2} = \sqrt{(5)^2 + (12)^2} = \sqrt{169} = 13 \text{ cm}$$

$\therefore$ Curved surface area of the cone $= \pi r l$

$$= \frac{22}{7} \times 5 \times 13 = \frac{1430}{7} \text{ cm}^2$$ **[1/2]**

Curved surface area of the cylinder $= 2\pi r h$

$$= 2 \times \frac{22}{7} \times 5 \times 12 = \frac{2640}{7} \text{ cm}^2$$

and area of upper base of the cylinder $= \pi r^2$

$$= \frac{22}{7} \times 5 \times 5 = \frac{550}{7} \text{ cm}^2$$ **[1]**

Now, total surface area of the remaining solid

= Curved surface area of the cylinder

+ Curved surface area of the cone

+ Area of upper base of the cylinder

$$= \frac{2640}{7} + \frac{1430}{7} + \frac{550}{7} = \frac{4620}{7} = 660 \text{ cm}^2$$ **[1]**

Common Mistake

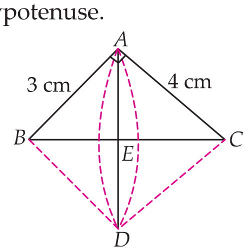
In finding slant height of the cone.

$$l = \sqrt{r^2 + h^2} = \sqrt{(5)^2 + (12)^2} = \sqrt{169} = \pm\ 13 \text{ cm}$$

But take always '+ ve' value because length or height can not be negative.

Or

(b) Let ABC be a right angled triangle, right angled at A and BC is the hypotenuse.



Also, let $AB = 3 \text{ cm}$ and $AC = 4 \text{ cm}$.

Then, $BC = \sqrt{3^2 + 4^2}$ [by Pythagoras theorem]

$= \sqrt{9 + 16} = \sqrt{25} = 5 \text{ cm}$ **[1]**

As, ΔABC revolves about the hypotenuse BC. It forms two cones ABD and ACD.

In ΔAEB and ΔCAB,

$\angle AEB = \angle CAB$ [each 90°]

$\angle ABE = \angle ABC$ [common]

$\therefore$ $\Delta AEB \sim \Delta CAB$ [by AA similarity criterion]

Then, $\frac{AE}{CA} = \frac{AB}{BC}$

[$\because$ in similar triangles, corresponding sides are proportional]

$\Rightarrow$ $\frac{AE}{4} = \frac{3}{5} \Rightarrow AE = \frac{12}{5} = 2.4 \text{ cm}$ **[1]**

So, radius of the base of each cone, $AE = 2.4$ cm

Now, in right angled ΔAEB,

$BE = \sqrt{AB^2 - AE^2}$ [by Pythagoras theorem]

$= \sqrt{(3)^2 - (2.4)^2} = \sqrt{9 - 5.76} = \sqrt{3.24} = 1.8 \text{ cm}$

So, height of the cone $ABD = BE = 1.8$ cm

$\therefore$ Height of the cone, $ACD = CE = BC - BE$

$= 5 - 1.8 = 3.2$ cm **[1]**

Now, volume of the cone ABD

$$= \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.4)^2 \times 1.8$$

$$= \frac{22}{21} \times 10.368 = 10.86 \text{ cm}^3$$

and volume of the cone ACD

$$= \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.4)^2 \times 3.2$$

[$\because$ radius will be same as AD is common]

$$= \frac{405.504}{21} = 19.31 \text{ cm}^3$$ **[1]**

Hence, required volume of double cone

$= 10.86 + 19.31 = 30.17 \text{ cm}^3$

Now, surface area of cone $ABD = \pi rl$

$= \frac{22}{7} \times 2.4 \times 3 = \frac{158.4}{7} = 22.63 \text{ cm}^2$

and surface area of cone $ACD = \pi rl$

$= \frac{22}{7} \times 2.4 \times 4 = \frac{211.2}{7} = 30.17 \text{ cm}^2$

Hence, required surface area of double cone

$= 22.63 + 30.17 = 52.8 \text{ cm}^2$ **[1]**

33. Given, Ram and Shyam together have 55 marbles.

Let Ram has x marbles.

Then, number of marbles Shyam has $= 55 - x$

$\because$ Both of them lost 5 marbles each.

$\therefore$ The number of marbles Ram has $= x - 5$

and the number of marbles Shyam has $= 55 - x - 5$

$= 50 - x$ **[1]**

Now, product of the number of marbles = 164

$\Rightarrow \quad (x-5)(50-x) = 164$

$\Rightarrow \quad 50x - x^2 - 250 + 5x = 164$

$\Rightarrow \quad -x^2 + 55x - 250 - 164 = 0$

$\Rightarrow \quad -x^2 + 55x - 414 = 0$

$\Rightarrow \quad x^2 - 55x + 414 = 0$ [$\because$ multiplying by (-1)] **[1]**

which is the required quadratic equation.

Now, by factorisation method, we get

$x^2 - 46x - 9x + 414 = 0$ **[1]**

$\Rightarrow \quad x(x-46) - 9(x-46) = 0$

$\Rightarrow \quad (x-46)(x-9) = 0$

$\Rightarrow \quad x - 46 = 0 \text{ or } x - 9 = 0$

$\Rightarrow \quad x = 46 \text{ or } x = 9$ **[1]**

When Ram has 46 marbles, then Shyam has

$= 55 - 46 = 9$ marbles

When Ram has 9 marbles, then Shyam has

$= 55 - 9 = 46$ marbles **[1]**

34. Table for the given data is

Class interval	Frequency (f_i)	Mid-value (x_i)	$d_i = x_i - A$	$f_i d_i$
0-20	17	10	−40	−680
20-40	f_1	30	−20	$-20 f_1$
40-60	32	$50 = A$	0	0
60-80	f_2	70	20	$20 f_2$
80-100	19	90	40	760
Total	$N = \Sigma f_i = 68 + f_1 + f_2$			$\Sigma f_i d_i = 80 - 20 f_1 + 20 f_2$

[2]

Here, $A = 50$

and $N = \Sigma f_i = 120$ [given]

$\Rightarrow \quad 68 + f_1 + f_2 = 120$

$\Rightarrow \quad f_1 + f_2 = 52$ ...(i) **[1]**

But mean = 50 [given]

$\therefore \quad A + \frac{\Sigma f_i d_i}{N} = 50$

$\Rightarrow \quad 50 + \frac{80 - 20 f_1 + 20 f_2}{120} = 50$ **[1]**

$\Rightarrow \quad 80 - 20 f_1 + 20 f_2 = 0 \Rightarrow 4 - f_1 + f_2 = 0$

$\Rightarrow \quad f_1 - f_2 = 4$ ...(ii)

On solving Eqs. (i) and (ii), we get

$f_1 = 28$ and $f_2 = 24$ **[1]**

Or

We have the mid-values, then firstly we should find the upper and lower limits of the various classes.
The difference between two consecutive values is $h = 125 - 115 = 10$.

Lower limit of a class = Mid-value $- h/2$

Upper limit = Mid-value $+ h/2$ **[1]**

Table for cumulative frequency is given below

Mid-value	Class groups	Frequency (f_i)	Cumulative frequency (cf)
115	110-120	6	6
125	120-130	25	31
135	130-140	48	79
145	140-150	72	151 (cf)
155	150-160	116 (f)	267
165	160-170	60	327
175	170-180	38	365
185	180-190	22	387
195	190-200	3	390
Total		$N = \Sigma f_i = 390$	

[2]

Here, $N = 390$

Now, $\frac{N}{2} = \frac{390}{2} = 195$

The cumulative frequency just greater than $N/2$ i.e. 195 is 267 and the corresponding class is 150-160. So, 150-160 is the median class.

Here, $l = 150$, $f = 116$, $h = 10$ and $cf = 151$ **[1]**

$\therefore$ Median $= l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h$

$= 150 + \left(\frac{195 - 151}{116} \right) \times 10$

$= 150 + \frac{44 \times 10}{116} = 150 + \frac{440}{116}$

$= 150 + 3.79 = 153.79$ **[1]**

35. **Given** $PQRS$ is a parallelogram, so $PQ \parallel SR$ and $PS \parallel QR$.

Also, $AB \parallel PS$

To prove $OC \parallel SR$

Proof In ΔOPS and ΔOAB,

$PS \parallel AB$ [given]

$\angle POS = \angle AOB$ [common angle]

$\angle OSP = \angle OBA$ [corresponding angles]

$\therefore \Delta OPS \sim \Delta OAB$ **[1]**

[by AA similarity criterion]

Then, $\frac{PS}{AB} = \frac{OS}{OB}$ ...(i)

[corresponding sides of similar triangles are proportional]

In ΔCQR and ΔCAB, $QR \parallel PS \parallel AB$

$\angle QCR = \angle ACB$ [common angle]

$\angle CRQ = \angle CBA$ [corresponding angle]

$\therefore \Delta CQR \sim \Delta CAB$ [by AA similarity criterion] **[1]**

Then, $\frac{QR}{AB} = \frac{CR}{CB}$ [corresponding sides of similar triangles are proportional]

$\Rightarrow \frac{PS}{AB} = \frac{CR}{CB}$ ...(ii)

[since, $PQRS$ is a parallelogram, so $PS = QR$]

From Eqs. (i) and (ii), we get

$\frac{OS}{OB} = \frac{CR}{CB}$ or $\frac{OB}{OS} = \frac{CB}{CR}$ **[1]**

On subtracting 1 from both sides, we get

$\frac{OB}{OS} - 1 = \frac{CB}{CR} - 1$

$\Rightarrow \frac{OB - OS}{OS} = \frac{CB - CR}{CR} \Rightarrow \frac{BS}{OS} = \frac{BR}{CR}$ **[1]**

By converse of basic proportionality theorem, we get

$SR \parallel OC$ **Hence proved.** **[1]**

36. Number of trees form a sequence 5, 8, 11, 14, ... which is an AP with first term, $a = 5$ and common difference, $d = 3$.

(i) We know that n^{th} term of AP is $a_n = a + (n-1)d$

$\therefore$ Number of trees in 10^{th} row

$= 5 + (10-1)3 = 5 + 27 = 32$

Hence, the number of trees in 10^{th} row is 32. **[1]**

(ii) Number of trees in 5^{th} row $= 5 + (5-1)3$

$[\because a_n = a + (n-1)d]$

$= 5 + 4 \times 3 = 17$

and number of trees in 6^{th} row $= 5 + (6-1)3$

$[\because a_n = a + (n-1)d]$

$= 5 + 5 \times 3 = 20$

$\therefore$ Ratio of number of trees in 5^{th} and 6^{th} row $= 17:20$ **[1]**

(iii) (a) Total number of trees in orchard after all 10 rows are planted

$S_{10} = \frac{10}{2}[2 \times 5 + (10-1)3]$ $\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$= 5[10 + 27] = 5 \times 37 = 185$

Hence, total number of trees in orchard after all 10 rows are planted is 185. **[1]**

Now, let number of rows be m

Then, $S_m = 258$

$\frac{m}{2}[2 \times 5 + (m-1)3] = 258$

$\Rightarrow m(10 + 3m - 3) = 516$

$\Rightarrow m(3m + 7) = 516$

$\Rightarrow 3m^2 + 7m - 516 = 0$

$\Rightarrow 3m^2 + 43m - 36m - 516 = 0$

$\Rightarrow (m + 43)(m - 12) = 0$

$\Rightarrow m = 12$ $[\because m \neq -43$, not possible$]$

Hence, required number of rows is 12. **[1]**

Or

(b) Since, Isha changed her plan.

So, there are two AP series forms.

First $AP_1 = 1^{st}, 2^{nd}, 3^{rd}, 4^{th}$ row

and second $AP_2 = 7^{th}, 8^{th}, 9^{th}, 10^{th}$ row

Total number of trees in $AP_1 = \frac{4}{2}[2 \times 5 + (4-1)3]$

$= 2[10 + 9] = 38$ **[1]**

Total number of trees in $AP_2 = \frac{4}{2}[2 \times 23 + (4-1)3]$

$= 2[46 + 9]$

$= 2 \times 55 = 110$

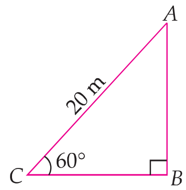
Hence, total number of trees after removing 5^{th} and 6^{th} rows

$= AP_1 + AP_2$

$= 38 + 110 = 148$ **[1]**

37. (i) Let in right angled ΔABC, AC is the rope and AB is a vertical pole.

Then, $AC = 20$ m, $\angle C = 60°$ and $\angle B = 90°$



In ΔABC, $\sin 60° = \frac{AB}{AC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AB}{20}$ $\left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$

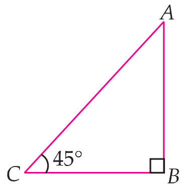
$\Rightarrow AB = 10\sqrt{3}$ m

Hence, height of the pole is $10\sqrt{3}$ m. **[1]**

(ii) (a) In ΔABC, $\angle C = 45°$

Then, $\sin 45° = \dfrac{AB}{AC}$ $\quad [\angle C = 45°]$

$\Rightarrow \quad \dfrac{1}{\sqrt{2}} = \dfrac{AB}{20} \quad \left[\because \sin 45° = \dfrac{1}{\sqrt{2}}\right]$



$\Rightarrow \quad \dfrac{20}{\sqrt{2}} = AB$

$\Rightarrow \quad AB = \dfrac{20}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$ [by rationalising]

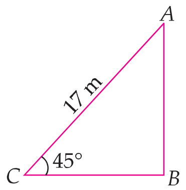
$= 10\sqrt{2}\,\text{m}$

Hence, the height of the pole is $10\sqrt{2}$ m. **[2]**

Or

(b) Since, 3 m rope is broken.

So, length of the rope $= 20 - 3 = 17$ m



$\sin 45° = \dfrac{AB}{AC} \quad [\because \angle C = 45°]$

$\Rightarrow \quad \dfrac{1}{\sqrt{2}} = \dfrac{AB}{17}$

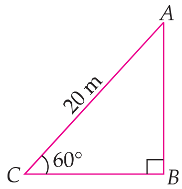
$\Rightarrow \quad AB = \dfrac{17}{\sqrt{2}}\,\text{m} \quad \left[\because \sin 45° = \dfrac{1}{\sqrt{2}}\right]$

$\Rightarrow \quad AB = \dfrac{17}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$\Rightarrow \quad AB = \dfrac{17\sqrt{2}}{2}\,\text{m}$

Hence, the height of pole will be $\dfrac{17\sqrt{2}}{2}$ m. **[2]**

(iii) In ΔABC, $\angle C = 60°$



Then, $\cos 60° = \dfrac{BC}{AC}$

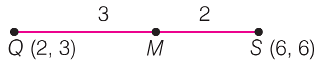
$\Rightarrow \quad \dfrac{1}{2} = \dfrac{BC}{20} \quad \left[\because \cos 60° = \dfrac{1}{2}\right]$

$\Rightarrow \quad BC = 10\,\text{m}$

Hence, the distance between artist and pole is 10 m. **[1]**

38. (i) The coordinates of points Q and S are (2, 3) and (6, 6). **[1]**

(ii) (a) By using internal division formula,



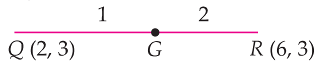
Coordinates of $M = \left(\dfrac{3\times6+2\times2}{3+2}, \dfrac{3\times6+2\times3}{3+2}\right)$

$= \left(\dfrac{18+4}{5}, \dfrac{18+6}{5}\right) = \left(\dfrac{22}{5}, \dfrac{24}{5}\right)$

Hence, coordinates of M are $\left(\dfrac{22}{5}, \dfrac{24}{5}\right)$. **[2]**

Or

(b) Using internal division formula,



Coordinates of $G = \left(\dfrac{1\times6+2\times2}{1+2}, \dfrac{1\times3+2\times3}{1+2}\right)$

$= \left(\dfrac{6+4}{3}, \dfrac{3+6}{3}\right) = \left(\dfrac{10}{3}, \dfrac{9}{3}\right) = \left(\dfrac{10}{3}, 3\right)$ **[2]**

(iii) Distance between the vertices of diagonal Q and S

$= \sqrt{(6-2)^2 + (6-3)^2} = \sqrt{4^2 + 3^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$ units **[1]**

My Reflection & Problem Points

Write down any difficulties, doubts, or mistakes you faced in this paper.

Discuss these points with your teacher and sort them out.

Concept (s) I got stuck on ....

....

Question (s) I couldn't complete ....

....

What confused me most ....

....

Time issue faced in ....